

# New String Theories in Six Dimensions via Branes at Orbifold Singularities

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We present several classes of new 6d string theories which arise via branes at orbifold singularities. They have compact moduli spaces, associated with tensor multiplets, given by Weyl alcoves of non-Abelian groups. We discuss T-duality and Matrix model applications upon compactification.

## 1. Introduction

It was recently pointed out in [1] that new 6d theories, which include stringy excitations but without gravity, can be obtained in the world-volume of five-branes by taking  $g_s \rightarrow 0$  with  $M_s$  held fixed. Four different classes were obtained in [1]:

- (iia) Theories with  $\mathcal{N} = (1, 1)$  supersymmetry, which are obtained in type IIB five-branes or, alternatively [2], via type IIA with a  $\mathbb{C}^2/\Gamma_G$  ALE singularity <sup>1</sup>.
- (iib) Theories with  $\mathcal{N} = (2, 0)$  supersymmetry, which are obtained in the world-volume of type IIA (or M-theory) five branes or, alternatively [2], via type IIB with a  $\mathbb{C}^2/\Gamma_G$  singularity.
- (o) Theories with  $\mathcal{N} = (1, 0)$  supersymmetry in the world-volume of  $SO(32)$  heterotic small-instantons or type I five-branes.
- (e) Theories with  $\mathcal{N} = (1, 0)$  supersymmetry in the world-volume of  $E_8$  small instantons. The (o) theory has a global  $SO(32)$  symmetry and the (e) theory has a global  $E_8 \times E_8$  symmetry.

The infrared limit of these theories, with energies small compared to  $M_s$ , appear to be local quantum field theories. In the (iib) and (e) cases these are non-trivial, interacting, RG fixed points, while the (iia) and (o) cases are IR free. Despite their different IR behavior, upon compactification to five-dimensions on a circle,  $T$  duality exchanges the (iia)  $\leftrightarrow$  (iib) and (o)  $\leftrightarrow$  (e) theories. Thus the full theories are *not* local quantum field theories [1].

In this paper, we discuss new 6d  $\mathcal{N} = (1, 0)$  theories associated with type II or heterotic five-branes at orbifold singularities in the orthogonal four dimensions. As in [1], we take  $g_s \rightarrow 0$  with  $M_s$  fixed. The fact that new theories could be thus obtained was also mentioned during the course of this work in a footnote in [3]. It was there pointed out that one could have a general, Ricci-flat, non-compact manifold  $\mathcal{M}_4$  in the remaining four directions, giving theories which, in principle, could depend on the uncountably infinite parameters needed to specify  $\mathcal{M}_4$ . However, as in [4], we expect most of these parameters are irrelevant in the  $g_s \rightarrow 0$  and that only the  $\mathbb{C}^2/\Gamma_G$  singularity type matters. Clearly the singularity itself can not be ignored; indeed, it breaks the supersymmetry of the (iia) or (iib) theories to  $\mathcal{N} = (1, 0)$  supersymmetry.

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<sup>1</sup> We label  $\Gamma_G \subset SU(2)$  using the well-known correspondence with the simply-laced groups  $G = A_r, D_r, E_{6,7,8}$ .

The 6d string theories which we present have compact “Coulomb branches,” associated with expectation values the scalar components of 6d  $\mathcal{N} = (1, 0)$  tensor multiplets, which are the “Coxeter boxes” (also referred to as the “Weyl alcove”) of non-Abelian groups. For any group  $\mathcal{G}$  of rank  $r$ , the Coxeter box is a compact subspace of  $\mathbb{R}^r$  given by all  $\vec{\Phi} \in \mathbb{R}^r$  which satisfy

$$\vec{\alpha}_\mu \cdot \vec{\Phi} + M_s^2 \delta_{\mu 0} \geq 0, \quad \mu = 0 \dots r, \quad (1.1)$$

where  $\vec{\alpha}_\mu$  are the simple roots, including the extended root  $\mu = 0$ , with  $\sum_{\mu=0}^r n_\mu \vec{\alpha}_\mu = 0$  ( $n_\mu$  are the Dynkin indices). The  $\mu \neq 0$  conditions in (1.1) give the non-compact Weyl chamber  $\mathbb{R}^r / \mathcal{W}_\mathcal{G}$ , where  $\mathcal{W}_\mathcal{G}$  is the Weyl-group. Including the  $\mu = 0$  condition gives the Coxeter box  $\mathbb{R}^r / \mathcal{C}_\mathcal{G} \cong (S^1)^r / \mathcal{W}_\mathcal{G}$ , where the Coxeter group  $\mathcal{C}_\mathcal{G}$  includes translations in the root lattice of  $\mathcal{G}$ . Compact Coxeter box moduli spaces, of size  $R^{-1}$ , also arise via Wilson loops upon reducing a  $\mathcal{G}$  gauge theory on a circle of radius  $R$ . We have written the size of the Coxeter box (1.1) as  $M_s^2$  because here it will be.

Coxeter boxes already appear in the theories (*iib*) and (*e*) mentioned above. Part of the moduli space of the (*iib*) theory obtained from  $K$  parallel five-branes is the  $U(K)$  Coxeter box of size  $M_s^2$ . The (*iib*) theory obtained from type IIB string theory on a  $\mathbb{C}^2 / \Gamma_G$  ALE singularity has, as part of its moduli space, the Coxeter box of size  $M_s^2$  of the corresponding *ADE* group  $G$ . The (*e*) theory obtained from  $K$  small  $E_8 \times E_8$  instanton five-branes has the Coxeter box, again of size  $M_s^2$ , for  $Sp(K)$  as its Coulomb branch.

We will simply note some basic features of the new 6d string theories, saving a more detailed analysis for further study. In the next section, we discuss theories associated with type IIB NS five-branes at orbifold singularities. The tensor multiplet moduli live on the Coxeter box of the simply laced group  $G$  associated with the singularity. In sect. 3 we discuss theories associated with  $SO(32)$  heterotic or type I branes at orbifold singularities. In these examples, the tensor multiplet moduli can live in the Coxeter box of a non-simply-laced subgroup of  $G$ . In sect. 4, we discuss theories associated with  $E_8 \times E_8$  branes at orbifold singularities. In sect. 5, we discuss  $T$  duality upon compactification. Finally, in sect. 6, we discuss applications of the theories to providing a definition of  $M$  theory on  $(ALE) \times T^5 \times \mathbb{R}^{1,1}$  and  $M$  theory on  $(ALE) \times (T^5 / \mathbb{Z}_2) \times \mathbb{R}^{1,1}$ .

## 2. Type IIB branes at a $\mathbb{C}^2/\Gamma_G$ orbifold singularity

For our first class of examples, consider  $K$  parallel type IIB NS five-branes at a  $\mathbb{C}^2/\Gamma_G$  orbifold singularity in the transverse directions. Having five-branes but no ALE singularity would lead to a *(iia)* theory of [1]. Having the ALE singularity but no five-branes would lead to a *(iib)* theory of [1]. Putting the two situations together leads to new  $\mathcal{N} = (1, 0)$  string theories, whose field theory infra-red limit was discussed in [5].

As discussed in [5], the  $\mathcal{N} = (1, 0)$  theory has gauge group

$$\prod_{\mu=0}^r U(Kn_\mu), \quad (2.1)$$

with matter multiplets in the representations  $\frac{1}{2} \oplus_{\mu\nu} a_{\mu\nu}(\square_\mu, \bar{\square}_\nu)$ . In addition, there are  $r \equiv \text{rank } G$  hyper-multiplets and tensor multiplets (which would give  $r$   $\mathcal{N} = (2, 0)$  matter multiplets for the theory with no five-branes).  $r$  of the  $U(1)$  factors in (2.1) have charged matter and are thus anomalous in 6d. As in [6,7], this means that these  $U(1)$  factors are spontaneously broken; they pair with the  $r$  hyper-multiplets mentioned above to get a mass. The massless, unbroken gauge group is thus

$$U(1) \times \prod_{\mu=0}^r SU(Kn_\mu), \quad (2.2)$$

with the  $U(1)$  factor decoupled, with no charged matter. Although the  $U(1)$  factors in (2.1) are massive, their  $D$  term equations still constrain the moduli space. Supersymmetry implies that the expectation values of the  $r$  hyper-multiplets involved in the  $U(1)$  anomaly cancelation appear as Fayet-Iliopoulos terms in these constraints [6]; these are the ALE blowing-up modes, which enter as background parameters in the 6d theory.

Taking  $g_s \rightarrow 0$  with  $M_s$  fixed, the tensor multiplet moduli space is the Coxeter box (1.1) of the corresponding  $ADE$  group  $G$ . This can be seen starting from the *(iib)* theory associated with the ALE space and no branes.

Using results found in [8,5] via anomalies, the effective gauge coupling of the  $r + 1$  gauge groups on the Coulomb branch can be written as

$$g_\mu^{-2}(\vec{\Phi}) = \vec{\alpha}_\mu \cdot \vec{\Phi} + M_s^2 \delta_{\mu 0} \quad (2.3)$$

where, as in (1.1), the  $\vec{\alpha}_\mu$  are the simple and extended roots of the  $ADE$  group  $G$  associated with the singularity. Using  $\vec{\alpha}_\mu \cdot \vec{\alpha}_\mu = \tilde{C}_{\mu\mu}$ , the extended Cartan matrix of  $G$ , the couplings

in (2.3) cancel the reducible  $\tilde{C}_{\mu\nu}\text{tr}F_\mu^2\text{tr}F_\nu^2$  anomaly terms found in [8,5]. We see that, as required, all  $g_\mu^{-2} \geq 0$  over the entire Coulomb box (1.1), with the various  $g_\mu^{-2} = 0$  along the boundaries of the Coulomb box. The “Landau pole” mentioned in [8,5] has been eliminated by the compactness of the Coulomb branch for finite  $M_s$ .

There is a Higgs mode of the theory corresponding to moving the  $K$  five-branes away from the  $X_G \cong \mathbb{C}^2/\Gamma_G$  ALE space. This Higgs branch moduli space is  $\mathcal{M}_H \cong (X_G)^K/S_K$ , as expected, with (2.2) broken to the diagonal  $U(K)_D$  away from the origin (or with non-zero Fayet-Iliopoulos parameters). This  $U(K)_D$  theory is the (iia) theory of the branes away from the singularity, with gauge coupling  $g_D^{-2} = \sum_{\mu=0}^r n_\mu g_\mu^{-2} = M_s^2$  as expected. The low energy theory has an enhanced, accidental  $\mathcal{N} = (1,1)$  supersymmetry which is not respected by the massive field theory and stringy modes.

There are also interesting new 6d theories associated with type IIA NS 5-branes at orbifold singularities, which require further understanding. For the case of  $K$  branes at a  $\mathbb{C}^2/\mathbb{Z}_M$  singularity, the 6d theory could be the same theory as that of  $M$  type IIB branes at a  $\mathbb{C}^2/\mathbb{Z}_K$  singularity (up to a decoupled tensor multiplet in the former and vector multiplet in the latter).

### 3. New theories from $SO(32)$ branes at ALE singularities

Our next class of new 6d string theories with  $\mathcal{N} = (1,0)$  supersymmetry arise from  $SO(32)$  heterotic or type I 5-branes at  $\mathbb{C}^2/\Gamma_G$  orbifold singularities. The low energy limit of these theories was discussed in [8,9,5] and also, via F-theory, in [10,11]. The gauge group is

$$\prod_{\mu \in \mathcal{R}} Sp(v_\mu) \times \prod_{\mu \in \mathcal{P}} SO(v_\mu) \times \prod_{\mu \in \mathcal{C}} U(v_\mu), \quad (3.1)$$

where the nodes of the extended  $G$  Dynkin diagram have been grouped into the sets  $\mathcal{R}$ ,  $\mathcal{P}$ ,  $\mathcal{C}$ ,  $\bar{\mathcal{C}}$  discussed in detail in [5]. As in the discussion following (2.1), the overall  $U(1)$  factor in each  $U(v_\mu)$  is anomalous and thus pairs with a hyper-multiplet to get a mass.

The tensor multiplet structure is related to the Coxeter box of the corresponding simply-laced group  $G$ , but modded out by a  $\mathbb{Z}_2$  action  $*$  which takes  $\mathcal{C} \leftrightarrow \bar{\mathcal{C}}$ . From the analysis in [9,5], the result is that the tensor multiplets for a  $\mathbb{C}^2/\Gamma_G$  singularity live in the Coxeter box of  $H \subset G$  with  $G \rightarrow H$  as

$$\begin{array}{ll} SU(2P) & \rightarrow Sp(P) \\ SO(4P+2) & \rightarrow SO(4P+1) \\ SO(4P) & \rightarrow SO(4P) \\ E_6 & \rightarrow F_4 \\ E_7 & \rightarrow E_7 \\ E_8 & \rightarrow E_8. \end{array} \quad (3.2)$$

The operation in (3.2) is the same modding out which appeared in the description of [12,13] for obtaining composite gauge invariance with non-simply-laced gauge groups. Although it is outside of the focus of this work, we note that the hyper-Kähler quotient construction of [8,5] for the moduli space of  $SO(N)$  instantons on ALE spaces suggests an interesting analog of the results of Nakajima. Briefly put, Nakajima [14] showed that  $\widehat{G}_N$  affine Lie algebras arise in analyzing the moduli space of  $U(N)$  instantons on  $\mathbb{C}^2/\Gamma_G$ . Similarly, we expect  $\widehat{H}_N$  affine Lie algebras to arise in analyzing the moduli space of  $SO(N)$  instantons on  $\mathbb{C}^2/\Gamma_G$ , with  $G \rightarrow H$  as in (3.2). The results of [14] find physical application, for example in [15], in showing that simply-laced composite gauge invariance is properly represented on massive modes. The conjectured appearance of  $\widehat{H}_N$  affine Lie algebras could find similar application in compactifications with non-simply-laced composite gauge invariance.

Other 6d theories can be obtained by making use of the fact, as in [7], that the gauge group of the heterotic or type I theory is actually  $Spin(32)/\mathbb{Z}_2$ . The low-energy limit of these string theories in the case of  $\mathbb{C}^2/\mathbb{Z}_{2P}$  singularities was discussed in [8], where it was (sloppily) referred to as the case without vector structure. The result is a theory based on the “type I5 quiver diagrams” of [6], with gauge group

$$\prod_{i=1}^P SU(v_\mu) \tag{3.3}$$

and tensor multiplets which live in the Coxeter box, of size  $M_s^2$ , of  $Sp(P-1)$ . For the simplest example,  $\mathbb{C}^2/\mathbb{Z}_2$ , the low energy theory is  $SU(2K)$  with two matter fields in the  $\square$  and sixteen in the  $\square$  and no tensor multiplet.

#### 4. New theories from $E_8 \times E_8$ branes at orbifold singularities

Our next class of new 6d string theories with  $\mathcal{N} = (1,0)$  supersymmetry arise via  $E_8 \times E_8$  5-branes at orbifold singularities in the  $g_s \rightarrow 0$  with  $M_s$  fixed limit. The gauge group and number of tensor multiplets associated with point-like  $E_8$  instantons at ADE orbifold singularities was obtained via F-theory in [11]. We take this opportunity to briefly spell out the massless matter content of these theories, which we determine from the results of [11] combined with anomaly considerations, as it was not presented in [11]. First, the irreducible  $\text{tr} F^4$  gauge anomalies must vanish; remaining reducible anomalies must then be canceled by coupling to the tensor multiplets. In addition, as discussed in [16], a  $\pi_6$

anomaly restricts  $SU(2)$  to have  $n_2 = 4 \bmod 6$ ,  $SU(3)$  to have  $n_3 = 0 \bmod 6$ , and  $G_2$  to have  $n_7 = 1 \bmod 3$ . A further general condition is

$$n_H - n_V + 29n_T = 30K + r, \quad (4.1)$$

where  $n_H$  is the total number of hyper-multiplets,  $n_V$  is the total number of vector multiplets,  $n_T$  is the number of tensor multiplets,  $K$  is the number of small instantons or five-branes, and  $r \equiv \text{rank} G$  is the number of ALE blowing-up modes. The condition (4.1) is a 6d analog of a 't Hooft anomaly matching condition for the gravitational anomaly.

The theory (e) for  $K$   $E_8 \times E_8$  five-branes and no singularity has a Coulomb branch with  $n_T = K$  tensor multiplets and no vector-multiplet gauge group. Putting the  $K$  5-branes at a  $\mathbb{C}^2/\mathbb{Z}_M$  singularity, with  $K \geq 2M$ , the result of [11] is that there is a Coulomb branch, again with  $n_T = K$  tensor multiplets, but with new gauge fields, with gauge group

$$SU(2) \otimes SU(3) \otimes \cdots \otimes SU(M-1) \otimes SU(M)^{\otimes(K-2M+1)} \otimes SU(M-1) \otimes \cdots \otimes SU(2). \quad (4.2)$$

The massless matter content consists of bi-fundamentals charged under each neighboring pair of gauge groups in (4.2) as well as an extra fundamental flavor for each of the two  $SU(2)$ s at the ends and for each of the two  $SU(M)$ s at the end of the string of  $SU(M)$ s. As remarked in [11], the gauge group in (4.2) agrees (up to replacing the  $SU(n)$  with  $U(n)$ ) with that of [17,18] which is mirror dual in three dimensions to  $U(M)$  gauge theory with  $K$  flavors; the above hyper-multiplet content also agrees with that of [17,18]. The theory with this gauge group and matter content is properly free of gauge anomalies (making use of couplings to  $K-3$  of the tensor multiplets to cancel the reducible gauge anomalies).

The theory with the above gauge group and matter content properly has a  $K+M-1$  dimensional Higgs branch, with the gauge group generically completely broken.  $M-1$  of the Higgs-branch moduli correspond to the blowing-up modes of the  $\mathbb{C}^2/\mathbb{Z}_M$  orbifold. The remaining  $K$  dimensions is the  $K$ -fold symmetric product of the ALE space with those  $M-1$  moduli, corresponding to the locations of the  $K$  identical, point-like instantons on the ALE space. For generic values of these moduli, the 5-branes are away from any singularity and there are no vector-multiplets; the gauge symmetry (4.2) is unHiggsed when the moduli are tuned, corresponding to putting the 5-branes on the singularity.

For  $K=6$  five-branes at a  $G=D_4$  singularity, the result of [11] is that the gauge group is  $SU(2) \times G_2 \times SU(2)$  with  $n_T = 6$  tensor multiplets. The matter content is determined by anomaly considerations to be  $\frac{1}{2}(\mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus \frac{1}{2}(\mathbf{2}, \mathbf{7}, \mathbf{1}) \oplus \frac{1}{2}(\mathbf{1}, \mathbf{7}, \mathbf{2}) \oplus \frac{1}{2}(\mathbf{1}, \mathbf{1}, \mathbf{2}) \oplus 2(\mathbf{1}, \mathbf{7}, \mathbf{1})$ . This

theory has a 10 dimensional Higgs branch, with the gauge group generically completely broken, corresponding to the location of the six point-like instantons on the ALE space and its four blowing-up modes. Giving an expectation value to a matter fields in the  $(\mathbf{1}, \mathbf{7}, \mathbf{1})$  corresponds to smoothing the  $D_4$  singularity to an  $A_2$  singularity.

For  $K \geq 7$  five-branes at a  $G = D_4$  singularity, the result of [11] is gauge group  $SU(2) \times G_2 \times SO(8)^{K-7} \otimes G_2 \otimes SU(2)$  with  $n_T = 2K - 7$ . The matter content is determined by anomaly considerations to be  $\frac{1}{2}(\mathbf{2}, \mathbf{1}) \oplus \frac{1}{2}(\mathbf{2}, \mathbf{7})$  for each  $SU(2) \times G_2$  pair and no other matter fields.

For  $E_6$ , the result of [11] is  $n_T = 4K - 22$ , with gauge group  $SU(2) \times G_2 \times F_4 \times G_2 \times SU(2)$  for  $K = 8$  and gauge group  $SU(2) \times G_2 \times F_4 \times SU(3) \times (E_6 \times SU(3))^{K-9} \times F_4 \times G_2 \times SU(2)$  for  $K > 8$ . The matter content is determined by anomaly considerations to consist, as above, of the minimal  $SU(2) \times G_2$  matter  $\frac{1}{2}(\mathbf{2}, \mathbf{1}) \oplus \frac{1}{2}(\mathbf{2}, \mathbf{7})$  in each pair of  $SU(2) \times G_2$ . For  $K = 8$  the  $F_4$  has a single matter field in the  $\mathbf{26}$  (giving it an expectation value breaks  $F_4 \rightarrow SO(9) \rightarrow SO(8)$ , corresponding to smoothing the singularity from  $E_6 \rightarrow D_5 \rightarrow D_4$ ). For  $K > 8$  each  $SU(2) \times G_2$  pair has the same minimal matter content as above, and there is no other matter.

For  $K \geq 10$  five-branes at a  $E_7$  singularity, the result of [11] is  $(SU(2) \times G_2)^4 \times F_4^2 \times E_7 \times (SU(2) \times SO(7) \times SU(2) \times E_7)^{K-10}$  with  $n_T = 6K - 40$ . Each  $SU(2) \times G_2$  factor has the minimal matter appearing above. Each  $SU(2) \times SO(7) \times SU(2) \times E_7$  factor has matter  $\frac{1}{2}(\mathbf{2}, \mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus \frac{1}{2}(\mathbf{1}, \mathbf{8}, \mathbf{2}, \mathbf{1})$ . There is no other matter.

For  $K \geq 10$  five-branes at a  $E_8$  singularity, the result of [11] is gauge group  $E_8^{(K-9)} \times F_4^{(K-8)} \times (SU(2) \times G_2)^{2K-16}$  with  $n_T = 12K - 96$ . Each  $SU(2) \times G_2$  factor has the minimal matter content appearing above and there is no other matter.

The result of [11] for  $K = 2m + 6$  five-branes at a  $D_{m+4}$  singularity is  $n_T = 2K - 6$  and gauge group  $SU(2) \times G_2 \times SO(9) \times SO(3) \times SO(11) \times SO(5) \times \cdots \times SO(2m + 5) \times SO(2m - 1) \times SO(2m + 7) \times SO(2m - 1) \times \cdots \times SO(9) \times G_2 \times SU(2)$ . For  $K > 2m + 8$  five-branes at a  $D_{m+1}$  singularity, [11] again find  $n_T = 2K - 6$  and, in addition to the gauge group factors for  $K = 2m + 6$ ,  $Sp(m) \times (SO(2m + 8) \times Sp(m))^{(K-2m-8)} \times SO(2m + 7)$ . For  $m > 1$ , we were not able to find a solution for matter content which is compatible with anomaly considerations and these gauge groups, though perhaps one does exist<sup>2</sup>.

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<sup>2</sup> **Note added** (in revised version, 9/3/97): There is a slight modification of the above gauge groups for which there is a matter content which is nicely compatible with all of the anomaly considerations. For  $K = 2m + 6$  five-branes at a  $D_{m+4}$  singularity, with  $n_T = 2K - 6$  as in [11],



## 5. Compactification and $T$ duality

It is natural to expect that, upon compactification on a circle, the new theories associated with five-branes at singularities are related by  $T$  duality, generalizing that of [1] between  $(iia) \leftrightarrow (iib)$  and  $(o) \leftrightarrow (e)$ . As in [1], this can be put to a simple test.

Upon compactifying on a circle, both the Cartan of the 6d gauge group and the 6d tensor multiplets lead to 5d  $U(1)$  gauge fields with scalar moduli. The number of 5d scalar moduli is thus  $r_V + n_T$ , where  $r_V$  is the rank of the 6d vector multiplet gauge group and  $n_T$  is the number of 6d tensor multiplets. Two 6d theories related by  $T$  duality must thus have  $r_V + n_T = \tilde{r}_V + \tilde{n}_T$ . More precisely, tensor multiplets in 6d have a compact “Coulomb branch,” with the scalar moduli living on a box of size  $M_s^2$ . Upon reducing to 5d and rescaling the modulus to have dimension one, it lives on a box of size  $M_s^2 R$ . On the other hand, reducing a 6d vector multiplet to 5d leads to a scalar modulus which lives on a box of size  $R^{-1}$ . Because  $T$  duality relates a theory compactified on a circle of radius  $R$  to another theory compactified on a circle of radius  $\tilde{R} \equiv (M_s^2 R)^{-1}$ , it exchanges 5d moduli associated with 6d tensor multiplets with those associated with 6d vector multiplets. Thus  $T$  dual theories must satisfy the stronger conditions  $\tilde{r}_V = n_T$  and  $\tilde{n}_T = r_V$ .

This can be thought of as a reason why, as we have seen, the Coulomb branch of 6d tensor multiplets is the Coxeter box of a non-Abelian group. Compactifying on a circle, there should be a  $T$  dual theory where these moduli *do* arise from a gauge theory with that gauge group.

For example, the vector multiplets of the  $(iia)$  theory compactified on a circle of radius  $R$  and the tensor multiplets of the  $(iib)$  theory compactified on a circle of radius

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the modified gauge group is  $SU(2) \times G_2 \times SO(9) \times Sp(1) \times SO(11) \times Sp(2) \times \cdots \times SO(2m + 5) \times Sp(m - 1) \times SO(2m + 7) \times Sp(m - 1) \times \cdots \times SO(9) \times G_2 \times SU(2)$ . The matter content which satisfies all of the anomaly equations is given by the minimal  $\frac{1}{2}((\mathbf{2}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{7}))$  in each  $SU(2) \times G_2$  factor and a half-hypermultiplet bi-fundamental charged under each neighboring  $SO$  and  $Sp$ , i.e. a  $\frac{1}{2}(\mathbf{2k} + \mathbf{7}, \mathbf{2k})$  under each neighboring  $SO(2k + 7) \times Sp(k)$  and a  $\frac{1}{2}(\mathbf{2k}, \mathbf{2k} + \mathbf{9})$  under each neighboring  $Sp(k) \times SO(2k + 9)$ . In addition, the middle  $SO(2m + 7)$  gauge group has a hypermultiplet in the  $\mathbf{2m} + \mathbf{7}$  which is uncharged under the other gauge groups. For the cases  $m = 2, 3$ , where the gauge group agrees with that of [11] (as  $Sp(1) \cong SO(3)$  and  $Sp(2) \cong SO(5)$ ), this matter content was first worked out by G. Rajesh. I am very grateful for his correspondence on the  $m = 2, 3$  cases, which helped to inspire the above modified gauge groups and matter content for  $m > 3$ . I also thank P.S. Aspinwall and D.R. Morrison for helpful correspondence on these issues. A similar modification of the gauge group and matter content applies for  $K > 2m + 8$ .

$\tilde{R} \equiv (M_s^2 R)^{-1}$  both lead to moduli living on a Coxeter box of size  $R^{-1}$ , compatible with their equivalence [1]. Similarly, both the  $SO(32)$  theory (*o*), compactified on a circle of radius  $R$ , with Wilson lines which break it to  $SO(16) \times SO(16)$ , and the theory (*e*) on a circle of radius  $\tilde{R} \equiv (M_s^2 R)^{-1}$  lead to a 5d moduli space which is the Coxeter box of  $Sp(K)$ , of size  $R^{-1}$ .

The theories associated with  $SO(32)$  and  $E_8 \times E_8$  branes at  $\mathbb{C}^2/\Gamma_G$  singularities do satisfy the condition  $r_V + n_T = \tilde{r}_V + \tilde{n}_T$ . Indeed, as also noted in [19], in both cases,  $r_V + n_T = C_2(G)K - |G|$ , where  $C_2(G)$  is the dual Coxeter number of the *ADE* group  $G$  and  $|G|$  is its dimension<sup>3</sup>. On the other hand, the two theories do not satisfy the stronger conditions  $\tilde{r}_V = n_T$  and  $\tilde{n}_T = r_V$ . It is not presently known how this failure should be interpreted or resolved.

## 6. Matrix Model Applications of the Theories.

Following [20], it was suggested in [21] that a M(atrix) description of  $M$  theory on  $X_G \times \mathbb{R}^{6,1}$ , where  $X_G$  is an ALE space asymptotic to  $\mathbb{C}^2/\Gamma_G$ , is given by quantum mechanics with 8 supersymmetries and gauge group  $\prod_{\mu=0}^r U(v_\mu)$  with matter  $\frac{1}{2} \oplus_{\mu\nu=0}^r a_{\mu\nu}(\square_\mu, \overline{\square}_\nu)$ . The (classical) moduli space of vacua of this theory for  $v_\mu = Kn_\mu$  is <sup>4</sup>  $(X_G \times \mathbb{R}^5)^K/S_K$ , corresponding to the location of  $K$  identical zero branes in the light-cone  $X_G \times \mathbb{R}^5$ . We propose a slight variant of this conjecture.

Now consider  $M$  theory on  $X_G \times T^5 \times R^{1,1}$ . Following [1], it is expected<sup>5</sup> that a definition of this theory is given by compactifying the new 6d theory of sect. 2 on a  $\widehat{T}^5$ . As

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<sup>3</sup> As also noted in [19], this agrees with the dimension (in hyper-multiplet units) of the moduli space of  $K$   $G$  instantons on  $K3$ . Duality between the heterotic theory on  $T^3$  and  $M$  theory on  $K3$  suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a  $T^3$  *actually is* the moduli space of  $K$   $G$  instantons on  $K3$ . Similarly, compactifying the theory of sect. 2 associated with type II branes at orbifold singularities, the dimension of the Coulomb branch is  $C_2(G)K$ . Duality between type II on a  $T^3$  and  $M$  theory on  $T^4$  suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a  $T^3$  is the moduli space of  $K$   $G$  instantons on  $T^4$ .

<sup>4</sup> This is the moduli space for generic Higgs expectation values. There is a larger Coulomb branch, of dimension  $5KC_2(G)$ , at the origin.

<sup>5</sup> I thank N. Seiberg for suggesting this.

in [1], there are 25 compactification parameters living in  $SO(5, 5, \mathbb{Z}) \backslash SO(5, 5) / (SO(5) \times SO(5))$ . Taking a rectangular torus with no  $B$  field,  $\widehat{T}^5$  is related to  $T^5$  as in [1], by:

$$\begin{aligned}\widehat{L}_i &= \frac{l_p^3}{RL_i} \\ M_s^2 &= \frac{R^2 L_1 L_2 L_3 L_4 L_5}{l_p^9},\end{aligned}\tag{6.1}$$

where  $R$  is the radius of the longitudinal direction and  $l_p$  is the eleven-dimensional Planck-length. Indeed, this gives the correct light-cone  $X_G \times T^5$  space-time from the moduli space of vacua (subject to the same discussion about the situation at the quantum level as in [22,1]).

In the limit of large  $T^5$ , this reduces to a slight variant of the suggestion of [21] outlined above. The massless gauge group of the 6d theory is given by (2.2) rather than  $\prod_{\mu=0}^r U(v_\mu)$ ; in addition, there are the  $n_T = r$  tensor multiplets. Upon compactification, the tensor multiplets yield  $U(1)^r$  gauge fields, the same number which became massive because of the anomaly. It is thus tempting to conclude that, upon compactification, the tensor multiplets simply give back the same  $U(1)$  factors which became massive in 6d because of the anomaly, giving back the original  $\prod_{\mu} U(Kn_\mu)$  theory in lower dimensions. However, this does not seem to be the case. The difference is that the matter fields  $\frac{1}{2} \oplus_{\mu\nu=0}^r a_{\mu\nu}(\square_\mu, \overline{\square}_\nu)$  were charged under the  $U(1)^r$  which became massive because of the 6d anomaly. On the other hand, these matter fields are neutral under the  $U(1)^r$  which the tensor multiplets give back upon compactification; the new  $U(1)^r$  has no charged matter. Taking the limit of large  $T^5$  in (6.1) thus yields a slight variant of the gauge theory of [21].

Following [1], we similarly expect that the 6d string theory from  $SO(32)$  or  $E_8 \times E_8$  heterotic five-branes at a  $X_G$  singularity, when compactified on  $\widehat{T}^5$  (which depends on the 105 parameters in  $SO(21, 5, \mathbb{Z}) \backslash SO(21, 5) / (SO(21) \times SO(5))$ ), gives a definition of  $M$  theory on  $X_G \times (T^5/\mathbb{Z}_2) \times \mathbb{R}^{1,1}$ .

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